

Chapter 21

Mapping QTL with Epistatic Effects and QTL-by-Environment Interaction Effects

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Purpose

To map quantitative trait loci (QTL) with additive, epistatic, and QTL-by-environment interaction effects for doubled haploid (DH) or recombinant inbred line (RIL) populations.

Definitions

Genetic Model

If multiple-environment data of DH or RIL populations are used for mapping QTL, the phenotypic value of the j th genetic entry in environment h can be expressed as shown in the genetic model

$$\begin{aligned}
 y_{hj} = & \mu + a_1 x_{A_{1j}} + a_2 x_{A_{2j}} + aa x_{AA_j} \\
 & + u_{E_N} e_{E_h} + u_{A_1 E_N} e_{A_1 E_h} + u_{A_2 E_N} e_{A_2 E_h} + u_{AAE_N} e_{AAE_h} \\
 & + \sum_f u_{M_{\beta}} e_{M_f} + \sum_l u_{MM_{ij}} e_{MM_l} + \sum_p u_{ME_{hpj}} e_{ME_{Np}} + \sum_q u_{MME_{hqi}} e_{MME_{Nq}} + \epsilon_{hj}
 \end{aligned}$$

where μ is the population mean; a_1 and a_2 are the additive effects of loci Q_1 and Q_2 , respectively; aa is the additive \times additive epistatic effect of loci Q_1 and Q_2 ; $x_{A_{1j}}$, $x_{A_{2j}}$, and x_{AA_j} are coefficients of these genetic main effects; e_{E_h} is the random effect of environment h with coefficient u_{E_N} ; $e_{A_1 E_h}$ (or $e_{A_2 E_h}$) is the additive \times environment interaction effect with coefficient $u_{A_1 E_N}$

(or $u_{A_2E_{hj}}$) for Q_1 (or Q_2); e_{AAE_h} is the epistasis \times environment interaction effect with coefficient $u_{AAE_{hj}}$; e_{M_f} is the marker main effect with coefficient u_{M_f} ; e_{MM_i} is the marker \times marker interaction effect with coefficient u_{MM_i} ; $e_{ME_{hp}}$ is the marker \times environment interaction effect with coefficient $u_{ME_{hpj}}$; $e_{MME_{hq}}$ is the marker \times marker \times environment interaction effect with coefficient $u_{MME_{hqj}}$; and ε_{hj} is the residual effect.

Mixed Linear Model

The epistatic QTL model can be expressed in matrix form as

$$\begin{aligned} \mathbf{y} &= \mathbf{X}\mathbf{b} + \mathbf{U}_E \mathbf{e}_E + \mathbf{U}_{A_1E} \mathbf{e}_{A_1E} + \mathbf{U}_{A_2E} \mathbf{e}_{A_2E} + \mathbf{U}_{AAE} \mathbf{e}_{AAE} \\ &\quad + \mathbf{U}_M \mathbf{e}_M + \mathbf{U}_{MM} \mathbf{e}_{MM} + \mathbf{U}_{ME} \mathbf{e}_{ME} + \mathbf{U}_{MME} \mathbf{e}_{MME} + \mathbf{e}_\varepsilon \\ &= \mathbf{X}\mathbf{b} + \sum_{u=1}^9 \mathbf{U}_u \mathbf{e}_u \\ &\sim N(\mathbf{X}\mathbf{b}, \mathbf{V} = \sum_{u=1}^9 \sigma_u^2 \mathbf{U}_u \mathbf{R}_u \mathbf{U}_u^T) \end{aligned}$$

where \mathbf{y} is the phenotype vector; \mathbf{b} is the fixed parameter vector for population mean and QTL effects; \mathbf{X} is the known incidence matrix of the fixed parameters; $\mathbf{e}_1 = \mathbf{e}_E \sim N(0, \sigma_E^2 \mathbf{I})$ is the vector of environment effects; $\mathbf{e}_2 = \mathbf{e}_{A_1E} \sim N(0, \sigma_{A_1E}^2 \mathbf{I})$ is the vector of $A_1 \times E$ interaction effects; $\mathbf{e}_3 = \mathbf{e}_{A_2E} \sim N(0, \sigma_{A_2E}^2 \mathbf{I})$ is the vector of $A_2 \times E$ interaction effects; $\mathbf{e}_4 = \mathbf{e}_{AAE} \sim N(0, \sigma_{AAE}^2 \mathbf{R}_{AAE})$ is the vector of $AA \times E$ interaction effects; $\mathbf{e}_5 = \mathbf{e}_M \sim N(0, \sigma_M^2 \mathbf{R}_M)$ is the vector of marker main effects; $\mathbf{e}_6 = \mathbf{e}_{MM} \sim N(0, \sigma_{MM}^2 \mathbf{R}_{MM})$ is the vector of interaction marker main effects; $\mathbf{e}_7 = \mathbf{e}_{ME} \sim N(0, \sigma_{ME}^2 \mathbf{R}_{ME})$ is the vector of $M \times E$ interaction effects; $\mathbf{e}_8 = \mathbf{e}_{MME} \sim N(0, \sigma_{MME}^2 \mathbf{R}_{MME})$ is the vector of $MM \times E$ interaction effects; $\mathbf{e}_9 = \mathbf{e}_\varepsilon \sim N(0, \sigma_\varepsilon^2 \mathbf{I})$ is the vector of residual effects; \mathbf{U}_u ($u=1, 2, \dots, 8$) is the known incidence matrix of the random effects, and $\mathbf{U}_9 = \mathbf{I}$.

Analysis Methodology

An approach of mixed-model-based composite interval mapping (MCIM) can be constructed for handling epistatic effects and QTL \times environment interaction effects. The likelihood function (L) for the parameters of fixed effects b and variance components $[\sigma_u^2]$ is

$$L(b, V) = (2\pi)^{-\frac{n}{2}} |V|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(y - Xb)^T V^{-1} (y - Xb)\right]$$

with the log of the likelihood function (l)

$$l(b, V) = -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln|V| - \frac{1}{2} (y - Xb)^T V^{-1} (y - Xb).$$

For searching QTL, the null hypothesis for genetic parameters (QTL main effects and Q \times E interaction effects) can be tested by the likelihood ratio statistic (LR):

$$LR = 2l_1(\hat{b}_1, v_1) - 2l_0(\hat{b}_0, v_0).$$

The maximum likelihood estimates of QTL effects in b can be obtained by

$$\hat{b} = (X^T V^{-1} X)^{-1} X^T V^{-1} y$$

with variance-covariance matrix

$$\text{var}(\hat{b}) = (X^T V^{-1} X)^{-1}.$$

Q \times E interaction effects (additive \times environment interaction $e_{A,E}$ and $e_{A_j E}$, epistasis \times environment interaction $e_{AA_j E}$) can be obtained by the best linear unbiased prediction (BLUP) method:

$$\hat{e}_u = \sigma_u^2 U_u^T Q y$$

with variance-covariance matrix

$$\text{var}(\hat{e}_u) = \sigma_u^4 U_u^T Q U_u$$

where $Q = V^{-1} - V^{-1}X(X^T V^{-1}X)^{-1}X^T V^{-1}$.

Originators

- Wang, D., Zhu, J., Li, Z.K., and Paterson, A.H. (1999). Mapping QTLs with epistatic effects and QTL \times environment interactions by mixed linear model approaches. *Theoretical and Applied Genetics* 99:1255-1264.
- Zhu, J. (1998). Mixed model approaches of mapping genes for complex quantitative traits. In Wang, L.Z. and Dai, J.R. (Eds.), *Proceedings of Genetics and Crop Breeding in China* (pp. 19-20). Chinese Agricultural Science and Technology Publication House, Beijing.
- Zhu, J. and Weir, B.S. (1998). Mixed model approaches for genetic analysis of quantitative traits. In Chen, L.S., Ruan, S.G., and Zhu, J. (Eds.), *Advanced Topics in Biomathematics: Proceedings of International Conference on Mathematical Biology* (pp. 321-330). World Scientific Publishing Co., Singapore.

Software Available

- Wang, D., Zhu, J., Li, Z.K., and Paterson, A.H. (1999). *User Manual for QTLMapper Version 1.0: A Computer Software for Mapping Quantitative Trait Loci (QTLs) with Additive Effects, Epistatic Effects and QTL \times Environment Interactions* (program free of charge). Contact Dr. Jun Zhu, Department of Agronomy, Zhejiang University, Hangzhou, China. E-mail: <jzhu@zju.edu.cn>.

EXAMPLE

Data of DH population with ninety-six lines and fifty-four markers on three chromosomes (provided by Drs. N. Huang and P. Wu). Data analysis method is described in detail in the user manual for QTLMapper Version 1.0 (Wang et al., 1999).

Data file (ckge.map) for map information:

```

_Chromosomes 3
_MarkerNumbers 18 15 21
_DistanceUnit cM

```

```

*MapBegin*
Marker#   ch1       ch2       ch3
1         0         0         0
2        19.236   12.9949   7.7618
3        16.2488   5.3402    13.2518
4         4.8552    22.2875   6.9239
5         4.8047    27.7327   9.8037

```


TraitBegin

Env#	Ind#	SH5 ;	Env#	Ind#	SH5 ;	Env#	Ind#	SH5 ;	Env#	Ind#	SH5 ;
1	1	52.5 ;	1	49	77.6 ;	2	1	43.8 ;	2	49	66.7 ;
1	2	62.5 ;	1	50	50.4 ;	2	2	39.5 ;	2	50	50.1 ;
1	3	77.9 ;	1	51	60.0 ;	2	3	57.8 ;	2	51	56.7 ;
1	4	57.2 ;	1	52	68.6 ;	2	4	44.9 ;	2	52	56.4 ;
1	5	51.7 ;	1	53	58.0 ;	2	5	41.9 ;	2	53	53.3 ;
1	6	62.5 ;	1	54	67.2 ;	2	6	44.2 ;	2	54	58.4 ;
1	7	56.0 ;	1	55	65.2 ;	2	7	46.8 ;	2	55	60.1 ;
1	8	62.7 ;	1	56	66.7 ;	2	8	51.4 ;	2	56	57.2 ;
1	9	62.1 ;	1	57	67.1 ;	2	9	46.4 ;	2	57	53.1 ;
1	10	76.2 ;	1	58	59.6 ;	2	10	65.6 ;	2	58	54.9 ;
1	11	69.1 ;	1	59	67.5 ;	2	11	53.0 ;	2	59	56.0 ;
1	12	68.4 ;	1	60	67.7 ;	2	12	58.4 ;	2	60	52.6 ;
1	13	45.4 ;	1	61	60.3 ;	2	13	40.2 ;	2	61	52.0 ;
1	14	68.4 ;	1	62	70.9 ;	2	14	59.1 ;	2	62	57.0 ;
1	15	83.9 ;	1	63	78.8 ;	2	15	67.7 ;	2	63	65.0 ;
1	16	81.5 ;	1	64	70.9 ;	2	16	67.7 ;	2	64	59.0 ;
1	17	74.4 ;	1	65	52.2 ;	2	17	63.7 ;	2	65	46.3 ;
1	18	73.9 ;	1	66	70.7 ;	2	18	68.0 ;	2	66	55.3 ;
1	19	58.7 ;	1	67	66.3 ;	2	19	48.7 ;	2	67	58.4 ;
1	20	64.5 ;	1	68	55.0 ;	2	20	51.8 ;	2	68	48.9 ;
1	21	61.2 ;	1	69	75.3 ;	2	21	49.9 ;	2	69	59.6 ;
1	22	48.5 ;	1	70	75.5 ;	2	22	41.1 ;	2	70	56.8 ;
1	23	48.2 ;	1	71	57.5 ;	2	23	34.1 ;	2	71	43.4 ;
1	24	83.5 ;	1	72	49.7 ;	2	24	71.4 ;	2	72	42.5 ;
1	25	55.2 ;	1	73	75.5 ;	2	25	44.6 ;	2	73	66.5 ;
1	26	49.6 ;	1	74	52.5 ;	2	26	47.0 ;	2	74	40.1 ;
1	27	77.3 ;	1	75	64.6 ;	2	27	62.3 ;	2	75	57.3 ;
1	28	78.5 ;	1	76	57.2 ;	2	28	60.8 ;	2	76	52.8 ;
1	29	71.0 ;	1	77	52.1 ;	2	29	64.1 ;	2	77	43.4 ;
1	30	67.2 ;	1	78	53.9 ;	2	30	62.3 ;	2	78	46.8 ;
1	31	80.9 ;	1	79	72.0 ;	2	31	67.6 ;	2	79	63.1 ;
1	32	88.4 ;	1	80	70.0 ;	2	32	77.3 ;	2	80	57.8 ;
1	33	79.2 ;	1	81	52.3 ;	2	33	66.1 ;	2	81	43.2 ;
1	34	77.1 ;	1	82	55.5 ;	2	34	58.6 ;	2	82	43.3 ;
1	35	72.7 ;	1	83	63.7 ;	2	35	57.9 ;	2	83	48.3 ;
1	36	78.6 ;	1	84	62.1 ;	2	36	60.3 ;	2	84	57.5 ;
1	37	65.1 ;	1	85	61.0 ;	2	37	55.1 ;	2	85	50.2 ;
1	38	48.8 ;	1	86	71.8 ;	2	38	44.3 ;	2	86	59.9 ;
1	39	65.5 ;	1	87	76.6 ;	2	39	47.1 ;	2	87	58.6 ;
1	40	79.3 ;	1	88	55.4 ;	2	40	71.4 ;	2	88	44.1 ;
1	41	81.8 ;	1	89	60.9 ;	2	41	70.1 ;	2	89	43.6 ;
1	42	63.8 ;	1	90	58.9 ;	2	42	57.4 ;	2	90	44.7 ;
1	43	96.6 ;	1	91	44.4 ;	2	43	76.0 ;	2	91	37.0 ;
1	44	67.2 ;	1	92	73.6 ;	2	44	56.7 ;	2	92	56.3 ;
1	45	55.5 ;	1	93	73.8 ;	2	45	46.1 ;	2	93	63.6 ;
1	46	44.2 ;	1	94	82.4 ;	2	46	28.3 ;	2	94	69.8 ;
1	47	80.1 ;	1	95	53.7 ;	2	47	67.7 ;	2	95	41.8 ;
1	48	75.9 ;	1	96	64.2 ;	2	48	66.7 ;	2	96	55.4 ;

TraitEnd

How to use the software:

1. Run QTLMAPPER.EXE to analyze QTL positions and effects. First create two files: one is a map file (ckge.map) and the other is a marker and trait file (ckge.txt). Choose run from submenu and map epistatic QTL.
2. After finishing the general analysis, choose output submenu and screen putative additive-effect QTL or epistatic QTL. The results are presented in Output 1.
3. Run jackknife test in output submenu for detecting significant additive and epistatic effects. The results are presented in Output 2.

Output 1 for Contribution of QTL Effects

```
// Result file created by QTLMapper V 1.0
// Data file name: D:\QTLSOURCE\ckge.txt
// Marker map file name: D:\QTLSOURCE\ckge.map
// Environments: yes
// Replications: no
// Contents: relative contributions (H^2) for putative main-effect
//           QTLs/epistatic QTLs
// Calculations based on: D:\QTLSOURCE\ckge.jke
// BGV control method: A (control marker main & interaction ef-
//           fects)
# Date: 2000-07-04 Time: 14:05:56
```

Trait 1: SH5

Ch-Ini	Int.Namei	Sitei(M)	Ch-Inj	Int.Namej	Sitej(M)	H^2(Ai)	H^2(Aj)	H^2(AAij)	H^2(AEi)	H^2(AEj)	H^2(AAEij)
1-5	M5-M6	0.00	1-17	M17-M18	0.04	0.0000	0.0714	0.0000	0.0000	0.0001	0.0000
1-9	M9-M10	0.00	1-15	M15-M16	0.12	0.0000	0.2532	0.0000	0.0000	0.0001	0.0011
2-6	M24-M25	0.28	3-18	M51-M52	0.06	0.0000	0.0727	0.0000	0.0000	0.0001	0.0000
2-9	M27-M28	0.02	3-16	M49-M50	0.00	0.0625	0.0000	0.0000	0.0000	0.0001	0.0000

General contributions:

Additive (A): $H^2(A) = 0.6131$; Epistasis: $H^2(AA) = 0.0000$
 QE Interactions: $H^2(AE) = 0.0003$; $H^2(AAE) = 0.0011$

End

Output 2 for QTL A and AA Effects

```
// Result file created by QTLMapper V 1.0
// Data file name: D:\QTLSOURCE\ckge.txt
// Marker map file name: D:\QTLSOURCE\ckge.map
// Environments: yes
// Replications: no
// Contents: Jackknife test results for epistatic QTLs
// Jackknife based on: D:\QTLSOURCE\ckge.file
// BGV control method: A (control marker main & interaction effects)
// Threshold probability: 0.005000
```

Date: 2000-07-04 Time: 13:48:12

Trait 1: SH5

